Fault-Tolerant Scheduling with Dynamic Number of Replicas in Heterogeneous Systems

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Outline

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  – Problem

• 2. System model

• 3. The MaxRe scheduling algorithm

• 4. Experiments

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1. Introduction

Fault tolerance: Active replication scheme\[^{[1-2]}\]:

The job will be completed even with maximum 2 failures occur.

1. Introduction

• In order to tolerate $t$ failures, the active replica scheme has to schedule $t+1$ processors for each task.

• The large resource redundancy problem:
  • Energy consumption
    – Electric power, etc.
    – According to EPA (Environmental Protection Agency) report 2007, Servers and data centers consumed 1.5 percent of total U.S. electricity consumption in 2006.
  • Economic cost
    – To store $x$ gigabytes data, it has to use $3x$ gigabytes storage space. Assuming the economic cost of each drive is $y$, the extra $2y$ cost is believed to be passed on to the customers eventually.
    – E.g. Facebook utilizes 4 replicas, and Google utilizes 3 replicas for each data volume.

• The problem is how to achieve the corresponding reliability with less resources (replicas).
1. Introduction

Our basic idea:
Combining the processors’ reliability analysis within the replica scheduling, and make use of less resources to achieve the 2-failures tolerating comparable reliability.

Tolerate 2 failures:

Active replication scheme  MaxRe scheduling algorithm

Reliability analysis
2. System Model

• Processor model: $P = \{p_0, p_1, p_2, \ldots, p_{m-1}\}$
  – The processor is fault free during its idle time;
  – The arrival of failures follows a Poisson distribution:
    $$f(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

• Job model:
  – A job is represented as a weighted directed acyclic graph (DAG): $G = (V, E)$
  – $n = |V|$ is the number of tasks.
  – $e = |E|$ is the number of edges.
2. System Model

Proposition 1:

When submitting a DAG-based workflow to a heterogeneous system, the more replicas for each task may not lead to a higher reliability.

Proof (one example):

\[(a): \frac{1}{2} \times \left(1 - \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{3}\right)\right) = 0.25\]

\[(b): 1 - \left(1 - \frac{1}{2} \times \frac{1}{4}\right) \times \left(1 - \frac{1}{9} \times \frac{1}{3}\right) \approx 0.1574\]
3. MaxRe scheduling algorithm

• Task priority\textsuperscript{[1]}:

\[
\text{rank}(\tau_i) = \overline{w}_i + \max_{\tau_j \in \text{succ}(\tau_i)} (\overline{c}_{i,j} + \text{rank}(\tau_j))
\]

Where:
- \text{rank}(t): priority value of task \( t \);
- \( \overline{w}_i \): the average computation cost of task \( t \);
- \text{succ}(t): the successor tasks of task \( t \);
- \( \overline{c}_{i,j} \): average communication cost from task \( t_i \) to task \( t_j \).

3. MaxRe scheduling algorithm

• For each task’s sub-reliability: \( r = \sqrt[n]{R} \)
  – Where \( r \) is the sub-reliability for each task;
  – \( n \) is the number of tasks;
  – \( R \) the overall reliability requirement.

• MaxRe:
3. MaxRe scheduling algorithm

Definition \( TT \) (Total Time):

\[
TT(p_j) = ET(\tau_i, p_j) + \sum_{\tau_k \in on(p_j)} ET(\tau_k, p_j)
\]

where \( ET(\tau_i, p_j) \) is the execution time when scheduling task \( \tau_i \) to processor \( p_j \). \( on(p_j) \) is the previous tasks that have already been scheduled on processor \( p_j \).

Definition \( CR \) (Current Reliability):

\[
CR(p_j) = e^{-\tilde{\lambda}_j TT[p_j]}
\]
3. MaxRe scheduling algorithm

The MaxRe scheduling:

- Task priority

Diagram showing the scheduling process with different tasks assigned to various priorities and resources.
3. MaxRe scheduling algorithm

• Analysis:

Suppose the reliability value provided by the MaxRe algorithm is \( R(\text{MaxRe}) \). When the required number of replicas for each task does not exceed the total number of processors, we have:
\[
R \leq R(\text{MaxRe}).
\]

Proof (key point):
\[
R = r^n \leq \prod_{i=0}^{n-1} 1 - \prod_{k=0}^{k<\xi} (1 - CR(\tau_i, p_k))
\]

\( CR(\tau_i, p_k) \) implies the probability that, for task \( i \), the replica \( p_k \) does not encounter failures from the very beginning to the \( i \)'s finish.
3. MaxRe scheduling algorithm

• The reliability of the MaxRe’s result:

Proposition 3:

The reliability $R(\text{MaxRe})$ provided by the MaxRe scheduling algorithm meets the following conditions:

$$R \leq R(\text{MaxRe}) \leq \prod_{i=1}^{i<n} (1 - \prod_{p_j \in \text{sche}(\tau_i)} (1 - R(\tau_i, p_j)))$$

Proof (key point):

$R(\tau_i, p_j)$ implies the probability that no failures occur during the time period that task $\tau_i$ is executed on processor $p_j$. 
4. Experiments Results

• The 4 workflow examples:

(a) Sample 1
(b) LQCD
(c) Stencil
(d) Doolittle

• The Parameters for the simulation platform

<table>
<thead>
<tr>
<th>Task</th>
<th>Processor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>Com_load</td>
</tr>
<tr>
<td>100 ~ 500</td>
<td>9 ~ 29</td>
</tr>
</tbody>
</table>

4. Experiment Results

• (1) The verification to Proposition 3:

\[ R = 0.93. \]

\[ R(\text{MaxRe}) > 0.96; \]

\[ R(\prod_{i=1}^{i<n} F'(\tau_i)) > 0.98 \]
4. Experiment Results

• (2) The resource usage with comparison to the FTSA algorithm

Resource usage metric:

\[
CUA = \sum_{i=0}^{n} \sum_{j=0}^{m} \left( ET(\tau_i, p_j) \times x_{ij} \right)
\]

(1) The MaxRe consumes at most 70% fewer resources than the FTSA algorithm.

(2) To tolerant more failures, FTSA algorithm needs more resources than the MaxRe algorithm.

4. Experiment Results

• (3) The execution time with comparison to the FTSA algorithm

The earliest finish time:
The earliest finish time of MaxRe’s schedule is at most 20 percent longer than FTSA.

The latest finish time:
The MaxRe scheduling algorithm even performs better than the FTSA Algorithm.
5. Conclusions and future works

Contributions:

We propose the MaxRe scheduling algorithm which satisfy the user’s reliability requirement with the minimum resources:

• (1) The theoretical analysis and experiments prove that MaxRe algorithm can satisfy the user’s reliability requirement.

• (2) The resource usage by MaxRe algorithm is at least 30 percent fewer than by FTSA algorithm (at most 70%).

• (3) The earliest finish time of MaxRe’s schedule is at most 20 percent longer than FTSA, while the latest finish time of MaxRe’s schedule is even shorter than FTSA.
5. Conclusion and future works

• The MaxRe algorithm is suitable when:
  – The user has special requirements on higher reliability, however, the resources are limited.

• Future works:
  – The consideration on the output schedule’s execution time;
  – The system performance analysis.
• Thank you for your attention,
• Questions?